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THE THEORY OF EFFECTIVE PROTECTION AND THE STOLPER-SAMUELSON THEOREM

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ABSTRACT

The purpose of the present paper is to demonstrate that, in the Bhagwati-Srinivasan general equilibrium model (1973) with many industries, many primary factors, many imported inputs and non-separable production functions, the weak Stolper-Samuelson theorem defined by Chipman (1969) is valid, if the factor inputs matrix is a P-matrix.

The views expressed here are the responsibility of the author and do not reflect those of the Department of Economics or the Massachusetts Institute of Technology.

The Theory of Effective Protection and the Stolper-Samuelson Theorem[1]

1. Introduction

Let us consider the general equilibrium model with n industries, n primary factors, m imported intermediate goods and non-separable production functions (that is, Bhagwati and Srinivasan's model (1973)). Suppose that protection is conferred on the i°th industry by the following change in the tariff structure:

$$\text{(I)} \qquad \hat{P}_{i^\circ} > \hat{P}^M_k = \hat{P}_j \qquad (k = 1,\ldots,m;\ j = 1,\ldots,i^\circ-1,i^\circ+1,\ldots,n)$$

where P_j and P^M_k denote the domestic prices of the jth good and the kth imported input, respectively and $\hat{z} \equiv dz/z$ for any variable z. Then we can ask: What will be the effects of the change in tariff structure (I) on domestic resource allocation? This problem which is at the heart of the theory of effective protection has recently received the attention of several economists. The studies of Bhagwati and Srinivasan (1973), Bruno (1973), Sendo (1974), Tanaka, Sendo and Kakimoto (1976) and Uekawa (1976) are particularly significant in this connection. The basic result due to Uekawa (1976) has established the following:

Proposition (A): For any given i° (= 1,...,n) if protection such as (I) is conferred on the i°th industry, then there will be a semi-positive resource inflow (at least one positive factor inflow and no factor

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withdrawal) into this industry and out of the rest of the economy under the following two conditions:

(i) There is gross substitutability between inputs in each production function, in the sense that the marginal product of any input does not decrease as the use of any other input is increased.

(ii) The rank of the primary factor inputs matrix is n.

The weak Stolper-Samuelson (WSS) theorem (Chipman 1969), extended to the present model involving imported intermediates, can be stated as follows:

<u>The weak Stolper-Samuelson Theorem:</u> For any given i° (= 1,...,n) if a change in the tariff structure such as (I) will take place, then the real price of the i°th factor will go up, that is, $\hat{W}_{i^\circ} > \hat{P}_{i^\circ}$.

In the strong Stolper-Samuelson theorem, we will have, in addition, $\hat{W}_j \leq \hat{P}_j$ for $j \neq i^\circ$.

It is well known that, in a two-industry-two factor model involving no imported inputs, both proposition (A) and the weak (and, in fact, the strong) Stolper-Samuelson theorem hold . It is interesting to ask in our general model involving more than two industries and permitting imported inputs: will the conditions (i) and (ii) that ensure the validity of proposition (A) be sufficient for the validity of WSS theorem? The answer is negative in general. The reason is the following. In the 2 x 2 model, condition (ii) implies a factor intensity ranking of the industries and this is what ensures the validity of the weak (and the strong) WSS criterion. But, in an n x n model, condition (ii) does not enable a factor intensity ranking of the industries defined in a suitable way, and some

condition to ensure such a ranking has/to be presumably assumed if the WSS theorem is to hold.

We may suspect, indeed, that the validity of the Stolper-Samuelson theorem will depend on some kind of factor intensity condition: for, Uekawa-Kemp-Wegge (1973) showed already that the weak Stolper-Samuelson theorem would be valid under the severe condition that the factor inputs matrix has the Minkowski property[2], without requiring gross substitutability between inputs. [On the other hand, we have so far assumed gross substitutability between inputs in ERP theory; this suggests that there could be some relationship between gross substitutability and one or more of the factor intensity conditions which have been so far considered by several economists in this literature.]

Thus, the purpose of this paper is to answer the following question: what condition would guarantee the validity of the weak Stolper-Samuelson theorem in the B-S model? The following principal result will be established in the present paper.

<u>If the factor inputs matrix is a P-matrix,[3] then the weak Stolper-Samuelson theorem is valid in the B-S model.</u>

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[2] Let $A^{-1} \equiv (a^{ij})$. Then the square matrix A is said to have the Minkowski property if it possesses an inverse with $a^{ii} > 0$ and $a^{ij} \leq 0$, $i \neq j$.

[3] The square matrix A is said to be a P-matrix if all of its principal minors are positive.

2. The ERP Theory and the Stolper-Samuelson Theorem

Let the production function for the ith good be $F^i(D^i, M^i)$ $(i = 1,\ldots,n)$, where $D^i = (D^i_1,\ldots,D^i_n)'$ is the column vector of domestic factor inputs and $M^i = (M^i_1,\ldots,M^i_m)'$ is the column vector of imported inputs.

We shall assume:

(A-1) All inputs enter into the production of each commodity.

(A-2) Each production function is homogeneous of degree one, concave and its Hessian matrix is indecomposable with negative diagonal and nonnegative off-diagonal elements (i.e., gross substitutability between inputs)

(A-3) Production takes place under perfect competition.

(A-4) Every commodity is produced

(A-5) The primary factor inputs matrix $D = (D^1,\ldots,D^n)$ is a P-matrix (i.e., a factor intensity condition).

Let $\bar{D} = (\bar{D}_1,\ldots,\bar{D}_n)'$ be the endowment vector of the primary factors in the economy and $W = (W_1,\ldots W_n)'$ a factor prices vector. Then given the domestic prices vector, $P = (P_1,\ldots,P_n)'$ and $P^M = (P^M_1,\ldots,P^M_m)'$, respectively of the outputs and imported inputs, the competitive equilibrium conditions are:

(1) $P_i F^i_D = W$ $\qquad i = 1,\ldots,n$

(2) $P_i F^i_M = P^M$ $\qquad i = 1,\ldots,n$

(3) $\sum_{i=1}^{n} D^i = \bar{D}$

where $F^i_D = (\partial F^i/\partial D^i_1,\ldots,\partial F^i/\partial D^i_n)'$ and $F^i_M = (\partial F^i/\partial M^i_1,\ldots,\partial F^i/\partial M^i_m)'$.

Differentiating equations (1) - (3) totally, we get

$$(4) \quad P_i[F^i_{DD}\dot{D}^i + F^i_{DM}\dot{M}^i] + \hat{P}_i W = \dot{W} \qquad i = 1,\ldots,n$$

$$(5) \quad P_i[F^i_{MD}\dot{D}^i + F^i_{MM}\dot{M}^i] + \hat{P}_i P^M = [\bar{P}^M]\hat{P}^M \qquad i = 1,\ldots,n$$

$$(6) \quad \sum_{i=1}^{n} \dot{D}^i = 0$$

where $\dot{z} \equiv dz$ for any variable, and $[\bar{v}]$ for any vector v is the diagonal matrix whose (i,i)th element is v_i, and $F^i_{MD} \equiv (\partial^2 F^i/\partial D^i_j \partial M^i_k) = (F^i_{DM})'$, $i,j = 1,\ldots,n$; $k = 1,\ldots,m$ and F^i_{DD} and F^i_{MM} are defined similarly as F^i_{MD}.

Eliminating $\dot{M}^i$ $(i = 1,\ldots,n)$, we get

$$(7) \quad A^i\dot{D}^i = -F^i_{DM}(F^i_{MM})^{-1}[\bar{P}^M](\hat{P}^M - \hat{P}_i e) - \hat{P}_i W + \dot{W} \qquad i = 1,\ldots,n$$

where $A^i \equiv P_i[F^i_{DD} - F^i_{DM}(F^i_{MM})^{-1}F^i_{MD}]$ is also negative semi-definite with negative diagonal and nonnegative off-diagonal elements.

In the change in the tariff structure (I), let $i° = 1$ without loss of generality. Then we have

$$(I)' \quad \hat{P}_1 > \hat{P}^M_k = \hat{P}_j = \sigma \qquad (k = 1,\ldots,m;\ j = 2,\ldots,n)$$

Thus we see from (I)' and (7) that

$$(8) \quad A^1\dot{D}^1 = -(\sigma - \hat{P}_1)F^1_{DM}(F^1_{MM})^{-1}P^M - \hat{P}_1 W + \dot{W}$$

$$(9) \quad A^i\dot{D}^i = -\sigma W + \dot{W} \qquad i = 2,\ldots,n$$

Now we shall show that the weak Stolper-Samuelson theorem is valid under assumptions (A-1) - (A-5). Let $A^1\dot{D}^1 = C = (C_i)$. Then it suffices to

show that $C_1 \geqq 0$. For we have from (8) that $\hat{W} = [\bar{W}]^{-1}C + (\sigma - \hat{P}_1)F^1_{DM}(F^1_{MM})^{-1}P^M + \hat{P}_1 e$ and therefore, $\hat{W}_1 - \hat{P}_1 = (C_1/W_1) + (\sigma - \hat{P}_1)(F^1_{DM}(F^1_{MM})^{-1}P^M)_1 > 0$, where $e = (1,\ldots,1)'$ is the sum vector.

Subtracting (9) from (8), we get

$$(10) \qquad A^1\dot{D}^1 = A^i\dot{D}^i - b \qquad\qquad i = 2,\ldots,n$$

where $b = (\hat{P}_1 - \sigma)[W - F^1_{DM}(F^1_{MM})^{-1}P^M] \geq 0$

Multiplying the ith equation of (10) by $(D^i)'$ and noting $(D^i)'A^i = 0$ $(i = 1,\ldots,n)$, we get

$$(11) \qquad D'C = -a$$

where $a = (0, (D^2)'b,\ldots,(D^n)'b)'$.

Thus, equation (11) satisfies the hypothesis of Theorem 2 of Appendix, since D is the positive P-matrix. Hence, we see that $C_1 > 0$. On the other hand, in my paper (1976) I showed that equations (10) and (6) have a solution such that $\dot{D}^1 > 0$.

Therefore, if protection is conferred on say, the first industry by the change in the tariff structure (I)', then the real price of the corresponding intensive factor will go up in the sense that $\hat{W}_1 > \hat{P}_1$, and at the same time, there will be a semi-positive resource inflow into this industry and out of the rest of the economy.

3. Appendix

Throughout this section we are concerned with a square matrix A of order n. The following notation is needed.

a_j the jth row vector of a matrix A

N the set $\{1,\ldots,n\}$

J a subset of N

$\bar{J}$ the complement of J relative to N.

I_J the diagonal matrix obtained from the identity matrix by replacing each jth row e_j by $-e_j$, $j \in J$

$A_{J\bar{J}}$ a submatrix of a matrix A which consists of the ith row and the jth column of A for $i \in J$ and $j \in \bar{J}$.

A(i) the matrix obtained from A by deleting the ith row vector

x_J a subvector of a vector x which consists of the jth component of x, $j \in J$.

ϕ the empty set

I will show that the following interesting two theorems on the P-matrix are valid.

Theorem 1: If a matrix A is a P-matrix, then for any proper subset J of N the system

(1) $x_J A_{JJ} - x_{\bar{J}} A_{\bar{J}J} > 0$

and

(2) $-x_J A_{J\bar{J}} + x_{\bar{J}} A_{\bar{J}\bar{J}} = 0$

has a non-zero solution $x = (x_J \quad x_{\bar{J}})$ such that $x_J > 0$

Proof:

Let $A = \begin{bmatrix} A_{JJ} & A_{J\bar{J}} \\ A_{\bar{J}J} & A_{\bar{J}\bar{J}} \end{bmatrix}$.

Then since

$$A^{-1} = \begin{bmatrix} [A_{JJ} - A_{J\bar{J}}(A_{\bar{J}\bar{J}})^{-1}A_{\bar{J}J}]^{-1} & -[A_{JJ} - A_{J\bar{J}}(A_{\bar{J}\bar{J}})^{-1}A_{\bar{J}J}]^{-1}A_{J\bar{J}}(A_{\bar{J}\bar{J}})^{-1} \\ -[A_{\bar{J}\bar{J}} - A_{\bar{J}J}(A_{JJ})^{-1}A_{J\bar{J}}]^{-1}A_{\bar{J}J}(A_{JJ})^{-1} & [A_{\bar{J}\bar{J}} - A_{\bar{J}J}(A_{JJ})^{-1}A_{J\bar{J}}]^{-1} \end{bmatrix}$$

is a P-matrix, $A_{JJ} - A_{J\bar{J}}(A_{\bar{J}\bar{J}})^{-1}A_{\bar{J}J}$ is also a P-matrix. Hence we have that the inequality

$$(3) \qquad x_J[A_{JJ} - A_{J\bar{J}}(A_{\bar{J}\bar{J}})^{-1}A_{\bar{J}J}] > 0$$

has a positive solution x_J.[4] Let $x_{\bar{J}} = x_J A_{J\bar{J}}(A_{\bar{J}\bar{J}})^{-1}$. Then we have

$$(4) \qquad x_J A_{JJ} - x_{\bar{J}} A_{\bar{J}J} = x_J[A_{JJ} - A_{J\bar{J}}(A_{\bar{J}\bar{J}})^{-1}A_{\bar{J}J}] > 0$$

and

$$(5) \qquad -x_J A_{J\bar{J}} + x_{\bar{J}} A_{\bar{J}\bar{J}} = -x_J A_{J\bar{J}} + x_J A_{J\bar{J}} = 0$$

Q.E.D.

---

[4] Let $x = (x_i)$ be a column vector and let $y = (y_i) = Ax$. Then A is said to reverse the sign of x if $x_i y_i \leqq 0$ for all i. A is a P-matrix if and only if A reverses the sign of no vector except zero. (See Gale and Nikaido (1965, Theorem2))

Now let A be a P-matrix. Suppose the inequality $yA > 0$ has no positive solution y. Then by duality the inequality $Ax \leqq 0$ has a semi-positive solution x. Hence, A reverses the sign of x and so A is not a P-matrix, a contradiction. Thus the inequality $yA > 0$ has a positive solution. See Gale (1960, p.49) on the duality theorem.

Theorem 2:[5] If a positive matrix A is a P-matrix, then the equation

(1) $Ax = -b$

has a solution x such that $x_1 \geqq 0$,

where $b = \begin{bmatrix} 0 \\ A(1) \end{bmatrix} v$ for any given positive vector v and $A(1) \equiv \begin{bmatrix} a_2 \\ \vdots \\ a_n \end{bmatrix}$

Proof: Since A is nonsingular, equation (1) has a unique non-zero solution. Let $J = \{j | x_j < 0\}$ and $\bar{J} = \{j | x_j \geqq 0\}$. Then $J \neq \phi$ and $\bar{J} \neq \phi$, since A is a positive matrix.

Suppose $1 \varepsilon J$. Then we have from (1) that the equation

(2) $[AI_J]z = -b$

has a nonnegative zolution z.

Since A is a P-matrix, we see from Theorem 1 that there exists a non-zero vector $y = (v_J v_{\bar{J}})$ such that

(3) $((v_J v_{\bar{J}})[I_J A I_J])_j > 0$ for $j \varepsilon J$

(4) $((y_J v_{\bar{J}})[I_J A I_J])_j = 0$ for $j \varepsilon \bar{J}$

(5) $v_J > 0$

Therefore we see from (3) - (5) that

(6) $(y_J y_{\bar{J}})\ I_J \begin{bmatrix} 0 \\ A(1) \end{bmatrix} > 0$

---

[5] Note that Theorem 2 is valid, even if A is a nonnegative P-matrix.

and so

$$(7) \qquad (yI_J)b = (v_J v_{\bar{J}})I_J \begin{bmatrix} 0 \\ A(1) \end{bmatrix} v > 0,$$

since $v > 0$. We see from (3), (4) and (7) that the inequality

$$(8) \qquad u\,[AI_J] \geqq 0 \text{ and } u\,b > 0$$

has a solution $u = yI_J$. Hence, by duality[6] equation (2) has no nonnegative solution, a contradiction. We must have $1 \in \bar{J}$ and therefore, equation (1) has a solution x such that $x_1 \geqq 0$.

Q.E.D.

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[6] Exactly one of the following alternatives holds. Either the equation $Ax = b$ has a nonnegative solution or the inequalities $vA \geqq 0$ and $vb < 0$ have a solution (see Gale (1960, p.44)).

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